Condition of perpendicularity of two lines:

$$\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$$

when $\theta = 90^\circ$; $l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$, condition of perpendicularity.

Condition of Parallelism: $\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$

$$\cos^2\theta = (l_1 l_2 + m_1 m_2 + n_1 n_2)^2$$

$$1 - \sin^2\theta = (l_1 l_2 + m_1 m_2 + n_1 n_2)^2$$

$$\therefore \sin^2\theta = 1 - (l_1 l_2 + m_1 m_2 + n_1 n_2)^2$$

$$= (l_1^2 + m_1^2 + n_1^2)(l_2^2 + m_2^2 + n_2^2) - (l_1 l_2 + m_1 m_2 + n_1 n_2)^2$$

After calculation; $\sin\theta = \pm\sqrt{(l_1 m_2 - l_2 m_1)^2 + (m_1 n_2 - m_2 n_1)^2 + (n_1 l_2 - n_2 l_1)^2}$

when $\theta = 0^\circ$; then $\sqrt{(l_1 m_2 - l_2 m_1)^2 + (m_1 n_2 - m_2 n_1)^2 + (n_1 l_2 - n_2 l_1)^2} = 0$

we get $l_1 = l_2$; $m_1 = m_2$; $n_1 = n_2$; which is the condition of Parallelism.

Q: Find the direction cosines of a line drawn from the origin to the point $(3, 2, -1)$.

Ans: Let $l, m, n$ be the direction cosines of the line OP.

then $l = (3 - 0)$; $3$, Similarly $m = 2 - 0 = 2$
$n = -1 - 0 = -1$


N.B: 1. We know that $l^2 + m^2 + n^2 = 1$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

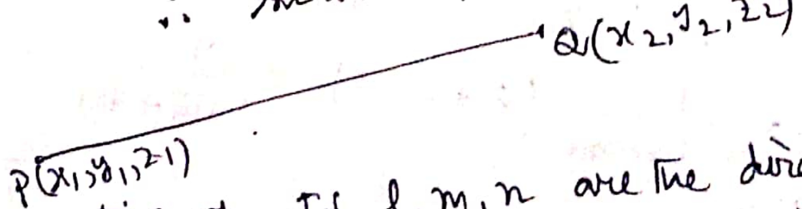
$$1 - \sin^2\alpha + 1 - \sin^2\beta + 1 - \sin^2\gamma = 1$$

$$3 - (\sin^2\alpha + \sin^2\beta + \sin^2\gamma) = 1$$

$$3 - 1 = 2.$$

$$\therefore \sin^2\alpha + \sin^2\beta + \sin^2\gamma$$

2.




~~Direction ratios of~~ If $l, m, n$ are the direction cosines of the line PQ; then $l = \frac{x_2 - x_1}{PQ}$; $m = \frac{y_2 - y_1}{PQ}$; $n = \frac{z_2 - z_1}{PQ}$

i.e $l \propto x_2 - x_1$; $m \propto y_2 - y_1$ & $n \propto z_2 - z_1$.

Q: Find the direction cosines of a line whose direction ratios are 3, 4, −2.

Ans :- Let the direction cosines of the line be $l, m, n$. given direction ratios are $3, 4, -2$; We know that

$$l = \pm\frac{a}{\sqrt{a^2+b^2+c^2}}, \quad m = \pm\frac{b}{\sqrt{a^2+b^2+c^2}}, \quad n = \pm\frac{c}{\sqrt{a^2+b^2+c^2}}$$

here $a = 3$, $b = 4$, $c = -2$.

$$\therefore l = \frac{3}{\pm\sqrt{3^2+4^2+(-2)^2}} = \frac{3}{\pm\sqrt{29}}, \quad \text{Similarly } m = \frac{4}{\pm\sqrt{29}}$$

$$\& \; n = -\frac{2}{\pm\sqrt{29}}.$$

Q: Find the angle between the lines whose directions ratios are 2, 3, −1 and 4, 1, 2.

A :- Let $l_1, m_1, n_1$ be the direction cosines of the line whose direction ratios are $2, 3, -1$; then

$$l_1 = \frac{2}{\pm\sqrt{2^2+3^2+(-1)^2}} = \frac{2}{\pm\sqrt{14}}, \quad \therefore m_1 = \frac{3}{\pm\sqrt{14}}, \quad n_1 = \frac{-1}{\pm\sqrt{14}}.$$

Let $l_2, m_2, n_2$ be the direction cosines of other lines whose direction ratios are $4, 1, 2$; then

$$l_2 = \frac{4}{\pm\sqrt{4^2+1^2+2^2}} = \frac{4}{\pm\sqrt{21}}; \quad m_2 = \frac{1}{\pm\sqrt{21}}, \quad n_2 = \frac{2}{\pm\sqrt{21}}.$$

If $\theta$ be the angle between the lines, then

$$\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$$

$$= \frac{2}{\pm\sqrt{14}}\cdot\frac{4}{\pm\sqrt{21}} + \frac{3}{\pm\sqrt{14}}\cdot\frac{1}{\pm\sqrt{21}} + \frac{-1}{\pm\sqrt{14}}\cdot\frac{2}{\pm\sqrt{21}}$$

$$= \frac{1}{\pm\sqrt{14}\cdot\pm\sqrt{21}}(8+3-2) = \frac{9}{\pm\sqrt{14}\pm\sqrt{21}} = \frac{3\times3}{\pm\sqrt{2}\cdot\sqrt{7}\cdot\sqrt{3}\cdot\sqrt{7}}$$

$$= \frac{3\times3}{\pm\sqrt{2}\sqrt{3}\cdot7}$$

$$= \pm\frac{3\sqrt{3}}{7\sqrt{2}}$$

$$\therefore \theta = \cos^{-1}\left(\pm\frac{3\sqrt{3}}{7\sqrt{2}}\right) \text{ Ans.}$$

31.Q: P-260. If the points P and Q are given by (2,3,4) and (1,1,−1) respectively, find the angle between OP and OQ.

Ans: The points $O(0,0,0)$, $P(2,3,4)$, $Q(1,1,-1)$.

Let $l_1, m_1, n_1$ be the dcs of OP.

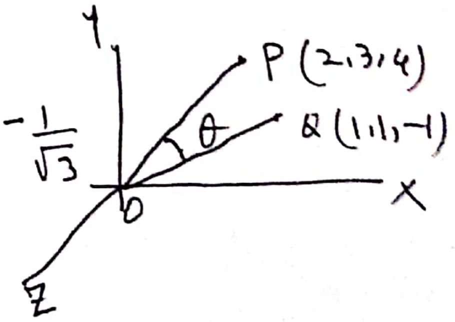
Then $l_1 = \frac{2}{\sqrt{29}}$, $m_1 = \frac{3}{\sqrt{29}}$, $n_1 = \frac{4}{\sqrt{29}}$

$l_1 \propto 2-0$
$l_1 = 2K$
$l_1^2 = 4K^2$
$\therefore m_1^2 = 9K^2$, $n_1^2 = 16K^2$
$\therefore l_1^2 + m_1^2 + n_1^2 = 29K^2$
$1 = 29K^2$
$K = \frac{1}{\sqrt{29}}$
$\therefore l_1 = \frac{2}{\sqrt{29}}$
Similarly, $m_1 = \frac{3}{\sqrt{29}}$
$n_1 = \frac{4}{\sqrt{29}}$

If $l_2, m_2, n_2$ are the dcs of OQ,

then $l_2 = \frac{1}{\sqrt{3}}$, $m_2 = \frac{1}{\sqrt{3}}$; $n_2 = \frac{-1}{\sqrt{3}}$

Let $\theta$ be the angle between the lines OP & OQ.




then
$$\cos\theta = l_1l_2 + m_1m_2 + n_1n_2$$
$$= \frac{2}{\sqrt{29}}\cdot\frac{1}{\sqrt{3}} + \frac{3}{\sqrt{29}}\cdot\frac{1}{\sqrt{3}} + \frac{4}{\sqrt{29}}\cdot\frac{-1}{\sqrt{3}}$$
$$= \frac{2+3-4}{\sqrt{3}\sqrt{29}}$$
$$= \frac{1}{\sqrt{3}\sqrt{29}}$$
$$\therefore \theta = \cos^{-1}\left(\frac{1}{\sqrt{3}\sqrt{29}}\right) \text{ Ans:}$$

Q. Find the direction cosines of the line joining the points $(3,2,-1)$ & $(4,3,2)$.

Ans: Let $P(3,2,-1)$ & $Q(4,3,2)$.

If $l, m, n$ are the direction cosines of PQ; then

$$l = \frac{4-3}{PQ} \;; \quad m = \frac{3-2}{PQ} \;; \quad n = \frac{2+1}{PQ}.$$

Now $PQ = \sqrt{(3-4)^2+(2-3)^2+(-1-2)^2}$

$= \sqrt{1+1+9} = \sqrt{11}$

$\therefore \; l = \frac{1}{\sqrt{11}}, \; m = \frac{1}{\sqrt{11}}, \; n = \frac{3}{\sqrt{11}}$ Ans.